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Numerical methods

MME308

Assignment 2

Grop.

Problem no: 3,7,13,14

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Problem (3)

Given:

$$2x_1 - 6x_2 - x_3 = -38$$

$$-3x_1 - x_2 + 7x_3 = -34$$

$$-8x_1 + x_2 - 2x_3 = 20$$

Required:

Use gauss elimination and gauss-Jordan elimination methods to solve the equations.

Solution:

1-by gauss elimination :

$$\begin{pmatrix} 2 & -6 & -1 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -38 \\ -34 \\ 20 \end{pmatrix}$$

- R1=R1/2

$$\begin{pmatrix} 1 & -3 & -0.5 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -19 \\ -34 \\ 20 \end{pmatrix}$$

- R2=R2+3R1
- R3=R3+8R1

$$\begin{pmatrix} 1 & -3 & -0.5 \\ 0 & -10 & 5.5 \\ 0 & -23 & -6 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -19 \\ -91 \\ -132 \end{pmatrix}$$

- R2=R2/-10

$$\begin{pmatrix} 1 & -3 & -0.5 \\ 0 & 1 & -0.55 \\ 0 & -23 & -6 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -19 \\ 9.1 \\ -132 \end{pmatrix}$$

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- R3=R3+23R2

$$\begin{pmatrix} 1 & -3 & -0.5 \\ 0 & 1 & -0.55 \\ 0 & 0 & -18.65 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -19 \\ 9.1 \\ 77.3 \end{pmatrix}$$

- R3=R3/-18.65

$$\begin{pmatrix} 1 & -3 & -0.5 \\ 0 & 1 & -0.55 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -19 \\ 9.1 \\ -4.1447 \end{pmatrix}$$

Now, this augmented matrix represents the equivalent linear system.

$$X_1 - 3X_2 - 0.5X_3 = -19 \qquad (1)$$
$$X_2 - 0.55X_3 = 9.1 \qquad (2)$$
$$X_3 = -4.1447 \qquad (3)$$

Since $X_3 = -4.1447$ from the last equation ,substituting in the equation (2) by X_3

$$X_2 - 0.55X3 = 9.1$$

$$X_2 - 0.55(-4.1447) = 9.1$$

That is, $\qquad\qquad\qquad$ $X_2 = 6.8204$

Substituting by X_3 ,X_2 in the equation (1).

$$X1 - 3X_2 - 0.5X3 = -1$$

$$X_1 - 3(6.8204) - 0.5(-4.1447) = -19$$

That is , $\qquad\qquad\qquad$ $X_1 = -0.6113$

Hence , the solution set consists of **x1=- *0.6113*   , x2=*6.8204*   , x3 =*-4.1447***

<u>Check answers in the original equation:</u> $\quad 2x_1 - 6x_2 - x_3 = -38$

$$2*(-0.6113) - 6*(6.8204) - (-4.1447) = -38.0003$$

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2-by gauss- Jordan elimination :

$$\begin{pmatrix} 2 & -6 & -1 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -38 \\ -34 \\ 20 \end{pmatrix}$$

- R1=R1/2

$$\begin{pmatrix} 1 & -3 & -0.5 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -19 \\ -34 \\ 20 \end{pmatrix}$$

- R2=R2+3R1
- R3=R3+8R1

$$\begin{pmatrix} 1 & -3 & -0.5 \\ 0 & -10 & 5.5 \\ 0 & -23 & -6 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -19 \\ -91 \\ -132 \end{pmatrix}$$

- R2=R2/-10

$$\begin{pmatrix} 1 & -3 & -0.5 \\ 0 & 1 & -0.55 \\ 0 & -23 & -6 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -19 \\ 9.1 \\ -132 \end{pmatrix}$$

- R3=R3+23R2

$$\begin{pmatrix} 1 & -3 & -0.5 \\ 0 & 1 & -0.55 \\ 0 & 0 & -18.65 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -19 \\ 9.1 \\ 77.3 \end{pmatrix}$$

- R3=R3/-18.65

$$\begin{pmatrix} 1 & -3 & -0.5 \\ 0 & 1 & -0.55 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -19 \\ 9.1 \\ -4.1447 \end{pmatrix}$$

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- R1=R1+3R2

- R2=R2+0.55R3

$$\begin{pmatrix} 1 & 0 & -2.15 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 8.3 \\ 6.8204 \\ -4.1447 \end{pmatrix}$$

- R1=R1+2.15R3

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.6113 \\ 6.8204 \\ -4.1447 \end{pmatrix}$$

Hence , the solution set consists of **x_1=- *0.6113* , x_2=*6.8204* , x_3=*-4.1447***

<u>Check answers in the original equation:</u> **$-3x_1 - x_2 + 7x_3 = -34$**

$-3x_1 - x_2 + 7x_3 = -34$

Problem (7)

Given:

$$0.3x_1 + 0.52x_2 + x_3 = -0.01$$

$$0.5x_1 + x_2 + 1.9x_3 = 0.67$$

$$0.1x_1 + 0.3x_2 + 0.5x_3 = -0.44$$

Required:

Use gauss elimination and gauss-Jordan elimination methods to solve the equations.

Solution:

1-by gauss elimination :

$$\begin{pmatrix} 0.3 & 0.52 & 1 \\ 0.5 & 1 & 1.9 \\ 0.1 & 0.3 & 0.5 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.01 \\ 0.67 \\ -0.44 \end{pmatrix}$$

- R1=R1/0.3

$$\begin{pmatrix} 1 & 1.7333 & 3.3333 \\ 0.5 & 1 & 1.9 \\ 0.1 & 0.3 & 0.5 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.0333 \\ 0.67 \\ -0.44 \end{pmatrix}$$

- R2=R2-0.5R1
- R3=R3-0.1R1

$$\begin{pmatrix} 1 & 1.7333 & 3.3333 \\ 0 & 0.1333 & 0.2333 \\ 0 & 0.1267 & 0.1667 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.0333 \\ 0.6667 \\ -0.4367 \end{pmatrix}$$

- R2=R2/0.1333

$$\begin{pmatrix} 1 & 1.7333 & 3.3333 \\ 0 & 1 & 1.75 \\ 0 & 0.1267 & 0.1667 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.0333 \\ 5.15 \\ -0.4367 \end{pmatrix}$$

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- R3=R3-0.1267R2

$$\begin{pmatrix} 1 & 1.7333 & 3.3333 \\ 0 & 1 & 1.75 \\ 0 & 0 & -0.055 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.0333 \\ 5.15 \\ -1.089 \end{pmatrix}$$

- R3=R3/-0.055

$$\begin{pmatrix} 1 & 1.7333 & 3.3333 \\ 0 & 1 & 1.75 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.0333 \\ 5.15 \\ 19.8 \end{pmatrix}$$

Now, this augmented matrix represents the equivalent linear system.

$$X_1 + 1.7333X_2 + 3.3333X_3 = -0.0333 \qquad (1)$$
$$X_2 + 1.75X_3 = 5.15 \qquad (2)$$
$$X_3 = 19.8 \qquad (3)$$

Since x3 = 19.8 from the last equation ,substituting in the equation (2) by x3

$$X2+1.75X3=5.15$$

$$X2+1.75(19.8)=5.15$$

That is,   *X2=-29.5*

Substituting by X_3 ,X_2 in the equation (1).

$$X1+ 1.7333X2+3.3333X3=-0.0333$$

$$X1+1.7333(-29.5)+3.3333(19.8)=-0.0333$$

That is ,   *X1=-14.9*

Hence , the solution set consists of x_1=-14.9 , x_2=-29.5 , x_3=19.8

Check answers in the original equation $0.3x_1 + 0.52x_2 + x_3 = -0.01$

$$0.3(-14.9) + 0.52(-29.5) +19.8 = -0.01$$

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1-by gauss-JORDAN elimination :

$$\begin{pmatrix} 0.3 & 0.52 & 1 \\ 0.5 & 1 & 1.9 \\ 0.1 & 0.3 & 0.5 \end{pmatrix}\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.01 \\ 0.67 \\ -0.44 \end{pmatrix}$$

- R1=R1/0.3

$$\begin{pmatrix} 1 & 1.7333 & 3.3333 \\ 0.5 & 1 & 1.9 \\ 0.1 & 0.3 & 0.5 \end{pmatrix}\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.0333 \\ 0.67 \\ -0.44 \end{pmatrix}$$

- R2=R2-0.5R1
- R3=R3-0.1R1

$$\begin{pmatrix} 1 & 1.7333 & 3.3333 \\ 0 & 0.1333 & 0.2333 \\ 0 & 0.1267 & 0.1667 \end{pmatrix}\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.0333 \\ 0.6667 \\ -0.4367 \end{pmatrix}$$

- R2=R2/0.1333

$$\begin{pmatrix} 1 & 1.7333 & 3.3333 \\ 0 & 1 & 1.75 \\ 0 & 0.1267 & 0.1667 \end{pmatrix}\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.0333 \\ 5.15 \\ -0.4367 \end{pmatrix}$$

- R3=R3-0.1267R2

$$\begin{pmatrix} 1 & 1.7333 & 3.3333 \\ 0 & 1 & 1.75 \\ 0 & 0 & -0.055 \end{pmatrix}\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.0333 \\ 5.15 \\ -1.089 \end{pmatrix}$$

- R3=R3/-0.055

$$\begin{pmatrix} 1 & 1.7333 & 3.3333 \\ 0 & 1 & 1.75 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -0.0333 \\ 5.15 \\ 19.8 \end{pmatrix}$$

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- R1=R1+1.7333*R2
- R2=R2+1.75*R3

$$\begin{pmatrix} 1 & 0 & 0.3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -8.96 \\ -29.5 \\ 19.8 \end{pmatrix}$$

- R1=R1+0.3R3

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} -14.9 \\ -29.5 \\ 19.8 \end{pmatrix}$$

Hence , the solution set consists of x_1=**-14.9** , x_2=**-29.5** , x_3=**19.8**

Check answers in the original equation $0.1x_1 + 0.3x_2 + 0.5x_3 = -0.44$

$0.1(-14.9) + 0.3(-29.5) + 0.5(19.8) = -0.44$

Problem 13

Given :

$$2x_1 + 8x_2 - x_3 = 11$$
$$5x_1 - x_2 + x_3 = 10$$
$$-x_1 + x_2 + 4x_3 = 3$$

Required :

Use gauss Seidel methods to solve the equations.

Solution:

$$X_1^{[K-1]} = \frac{b_1 - a_{12}X_2^{k} + a_{23}X_3^{k}}{a_{11}}$$

$$X_2^{[k-1]} = \frac{b_2 - a_{21}X_1^{[k-1]} + a_{23}X_3^{[k-1]}}{a_{11}}$$

$$X_3^{[k-1]} = \frac{b_3 - a_{31}X_1^{[k-1]} + a_{32}X_2^{[k-1]}}{a_{11}}$$

New Writing Eq's;

$$5x_1 - x_2 + x_3 = 10$$
$$2x_1 + 8x_2 - x_3 = 11$$
$$-x_1 + x_2 + 4x_3 = 3$$

at k=0

p0[0 , 0 , 0]

$$X_1^{[1]} = \frac{10 + (1)*X_2^{[0]} + (-1)*X_3^{[0]}}{5}$$

$$X_1^{[1]} = 2$$

$$X_2^{[1]} = \frac{11 + (2)*X_1^{[1]} + (1)*X_3^{[0]}}{8}$$

$$X_2^{[1]} = 0.875$$

$$X_3^{[1]} = \frac{3 + (1)*X_1^{[1]} + (-1)*X_2^{[1]}}{4}$$

$$X_3^{[1]} = 1.0313$$

$$\acute{\varepsilon}X_1 = \frac{2-0}{2} * 100 = 100$$

$$\acute{\varepsilon}X_2 = \frac{0.875-0}{0.875} * 100 = 100$$

$$\acute{\varepsilon}X_3 = \frac{1.0313-0}{1.0313} * 100 = 100$$

P1[2 , 0.875 , 1.0313] ,   at k=1

$$X_1^{[2]} = \frac{10 + (1)*X_2^{[1]} + (-1)*X_3^{[1]}}{5}$$

$$X_1^{[2]} = 2.3813$$

$$X_2^{[2]} = \frac{11 + (2)*X_1^{[2]} + (1)*X_3^{[1]}}{8}$$

$$X_2^{[2]} = .9086$$

$$X_3^{[2]} = \frac{3 + (1)*X_1^{[2]} + (-1)*X_2^{[2]}}{4}$$

$$X_3^{[2]} = 1.1182$$

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$$\acute{\epsilon}x_1 = \frac{2.3813 - 2}{2.3813} * 100 = 16.01$$

$$\acute{\epsilon}x_2 = \frac{0.9086 - 0.875}{0.9086} * 100 = 3.7$$

$$\acute{\epsilon}x_3 = \frac{1.1182 - 1.0313}{1.1182} * 100 = 7.77$$

P2[2.3813 , 0.9086, 1.1182]   at k=2

$$X_1^{[3]} = \frac{10 + (1)*X_2^{[2]} + (-1)*X_3^{[2]}}{5}$$

$$X_1^{[3]} = 2.4054$$

$$X_2^{[3]} = \frac{11 + (2)*X_1^{[3]} + (1)*X_3^{[2]}}{8}$$

$$X_2^{[3]} = 0.9134$$

$$X_3^{[3]} = \frac{3 + (1)*X_1^{[3]} + (-1)*X_2^{[3]}}{4}$$

$$X_3^{[3]} = 1.123$$

$$\acute{\epsilon}x_1 = \frac{2.4054 - 2.3813}{2.4054} * 100 = 1$$

$$\acute{\epsilon}x_2 = \frac{0.9134 - 0.9086}{0.9134} * 100 = 0.053$$

$$\acute{\epsilon}x_3 = \frac{1.123 - 1.1182}{1.123} * 100 = 0.43$$

K	X_1	X_2	X_3	E1	E2	E3
0	0	0	0			
1	2.0000	0.8750	1.0313	100	100	100
2	2.3813	0.9086	1.1182	16.01	3.70	7.77
3	2.4054	0.9134	1.1230	1.00	0.53	0.43
4	2.4073	0.9136	1.1234	0.08	0.01	0.04
5	2.4074	0.9136	1.1235	0.00	0.00	0.00

Since , error less than 10^4

Hence , the solution set consists of $x_1=2.4074, x_2=0.9136, x_3=1.1235$

<u>Check answers in the original equation</u>

$$-x_1 + x_2 + 4x_3 = 3$$

$$-(2.4074) + (0.9136) + 4(1.1235) = 3$$

Problem 14

Given :

$$x_1 - 5x_2 - x_3 = -8$$
$$4x_1 + x_2 - x_3 = 13$$
$$2x_1 - x_2 - 6x_3 = -2$$

Required :

Use gauss Seidel methods to solve the equations.

Solution:

$$X_1^{[K-1]} = \frac{b_1 - a_{12}X_2^{k} + a_{23}X_3^{k}}{a_{11}}$$

$$X_2^{[k-1]} = \frac{b_2 - a_{21}X_1^{[k-1]} + a_{23}X_3^{[k-1]}}{a_{11}}$$

$$X_3^{[k-1]} = \frac{b_3 - a_{31}X_1^{[k-1]} + a_{32}X_2^{[k-1]}}{a_{11}}$$

New Writing Eq's;

$$4 x_1 + x_2 - x_3 = 13$$
$$x_1 - 5x_2 - x_3 = -8$$
$$2x_1 - x_2 - 6x_3 = -2$$

at k=0

p0[0 , 0 , 0]

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$$X_1^{[1]} = \frac{10 + (1)*X_2^{[0]} + (-1)*X_3^{[0]}}{4}$$

$X_1^{[1]} = 3.25$

$$X_2^{[1]} = \frac{13 + (2)*X_1^{[1]} + (1)*X_3^{[0]}}{5}$$

$X_2^{[1]} = 2.25$

$$X_3^{[1]} = \frac{3 + (1)*X_1^{[1]} + (-1)*X_2^{[1]}}{6}$$

$X_3^{[1]} = 1.0417$

$$\acute{\varepsilon}X_1 = \frac{3.25 - 0}{3.25} * 100 = 100$$

$$\acute{\varepsilon}X_2 = \frac{2.25 - 0}{2.25} * 100 = 100$$

$$\acute{\varepsilon}X_3 = \frac{1.0417 - 0}{1.0417} * 100 = 100$$

P1[3.25 , 2.25 , 1.0417] ,   at k=1

$$X_1^{[2]} = \frac{10 + (1)*X_2^{[1]} + (-1)*X_3^{[1]}}{5}$$

$X_1^{[2]} = 2.94790$

$$X_2^{[2]} = \frac{13 + (2)*X_1^{[2]} + (1)*X_3^{[1]}}{8}$$

$X_2^{[2]} = 1.9813$

$$X_3^{[2]} = \frac{3 + (1)*X_1^{[2]} + (-1)*X_2^{[2]}}{4}$$

$X_3^{[2]} = 0.9858$

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$$\acute{\epsilon}x_1 = \frac{2.9479 - 3.25}{2.9479} * 100 = 10.24$$

$$\acute{\epsilon}x_2 = \frac{1.9813 - 2.25}{1.9813} * 100 = 13.56$$

$$\acute{\epsilon}x_3 = \frac{0.9858 - 1.0417}{0.9858} * 100 = 5.67$$

P2[2.9479, 1.9813, 0.9858]   at k=2

$$X_1^{[3]} = \frac{10 + (1)*X_2^{[2]} + (-1)*X_3^{[2]}}{5}$$

$X_1^{[3]} = 3.0011$

$$X_2^{[3]} = \frac{13 + (2)*X_1^{[3]} + (1)*X_3^{[2]}}{8}$$

$X_2^{[3]} = 2.0031$

$$X_3^{[3]} = \frac{3 + (1)*X_1^{[3]} + (-1)*X_2^{[3]}}{4}$$

$X_3^{[3]} = 0.9999$

$$\acute{\epsilon}x_1 = \frac{3.0011 - 2.9479}{3.0011} * 100 = 1.77$$

$$\acute{\epsilon}x_2 = \frac{2.0031 - 1.9813}{2.0031} * 100 = 1.09$$

$$\acute{\epsilon}x_3 = \frac{0.9999 - 0.9858}{0.9999} * 100 = 1.41$$

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i	X_1	X_2	X_3	Ex_1	Ex_2	Ex_3
0	0.0000	0.0000	0.0000			
1	3.2500	2.2500	1.0417	100	100	100
2	2.9479	1.9813	0.9858	10.2473	13.5647	5.6710
3	3.0011	2.0031	0.9999	1.7731	1.0895	1.4102
4	2.9992	1.9999	0.9998	0.0644	-0.1603	0.0109
5	3.0000	2.0000	1.0000	0.0258	0.0088	0.0229
6	3.0000	2.0000	1.0000	0.0004	-0.0022	0.0012

Since ,error less than 10^4

Hence , the solution set consists of $x_1=3$, $x_2=2$, $x_3=1$

<u>**Check answers in the original equation**</u>

$$4 x_1 + x_2 - x_3 = 13$$

$$4 (3) + (2) - (1) = 13$$